8.1 - Preliminary Theory–Linear Systems

Parallels exist between concepts, theorems, and vocabulary of linear differential equations and linear systems of differential equations:

- Existence and uniqueness of solutions
- The superposition principle
- Linear dependence/independence and how they relate to the Wronskian (though the Wronskian is different here)
- Fundamental set of solutions
- Homogeneous and nonhomogeneous systems
- Complementary and particular solutions

Recall from algebra: A system of equations can be represented as a matrix equation. For instance, the system

$$\begin{cases} 2x + 3y - 5z = 12 \\ 7x + 4y + 6z = 18 \\ -2x \qquad + \ z = -27 \end{cases}$$ can be represented by the matrix equation

In this chapter, we will primarily be concerned with linear systems of DEs that have the forms

$$\mathbf{X}' = \mathbf{AX} \qquad \text{and} \qquad \mathbf{X}' = \mathbf{AX} + \mathbf{F}$$

Example: Write the given linear system in matrix form.

$$\begin{cases} \dfrac{dx}{dt} = 4x - 7y \\ \dfrac{dy}{dt} = 5x \end{cases}$$

Example: Write the given linear system in matrix form.

$$\begin{cases} \dfrac{dx}{dt} = -3x + 4y + e^{-t}\sin 2t \\ \dfrac{dy}{dt} = 5x + 9z + 4e^{-t}\cos 2t \\ \dfrac{dz}{dt} = y + 6z - e^{-t} \end{cases}$$

Example: Write the given linear system without the use of matrices.

$$\mathbf{X}' = \begin{pmatrix} 7 & 5 & -9 \\ 4 & 1 & 1 \\ 0 & -2 & 3 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^{5t} - \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} e^{-2t}$$

Example: Verify that the vector **X** is a solution of the given homogeneous linear system.

$$\mathbf{X}' = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{X}; \qquad \mathbf{X} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{t} + \begin{pmatrix} 4 \\ -4 \end{pmatrix} te^{t}$$